

On the Accuracy of Embeddings for Internet Coordinate Systems

Eng Keong Lua, Timothy Griffin, Marcelo Pias, Han Zheng, Jon Crowcroft

University of Cambridge, Computer Laboratory

Email: {eng.keong-lua, timothy.griffin, marcelo.pias, han.zheng, jon.crowcroft}@cl.cam.ac.uk

Abstract

Internet coordinate systems embed Round-Trip-Times (RTTs) between Internet nodes into some geometric space so that unmeasured RTTs can be estimated using distance computation in that space. If accurate, such techniques would allow us to predict Internet RTTs without extensive measurements. The published techniques appear to work very well when accuracy is measured using metrics such as *absolute relative error*. Our main observation is that absolute relative error tells us very little about the *quality* of an embedding as experienced by a user. We define several new accuracy metrics that attempt to quantify various aspects of user-oriented *quality*. Evaluation of current Internet coordinate systems using our new metrics indicates that their quality is not as high as that suggested by the use of absolute relative error.

1 Introduction and Motivation

An Internet coordinate system starts with a collection of nodes and measured Round-Trip-Times (RTTs) between some pairs of these nodes. It then *embeds* the nodes into a geometric space by associating each node with a point in that space. The goals of such an embedding are twofold. First, the embedding should be *accurate* in the sense that the geometric distance between embedded nodes should, in some sense, closely approximate their RTTs. Second, the methods should remain accurate even when the input is a small subset of all possible RTT measurements. That is, a node’s coordinates should allow even the unmeasured RTTs to be estimated accurately.

Many Internet coordinate systems have been described in the literature. Accuracy is typically assessed using metrics similar to *absolute relative error*, which forms an average over every pair of nodes of the absolute value of the difference between their embedded distance and their original distance. Note that an embedding technique does not require a full mesh of RTT measurements, but assessing its

accuracy surely does. The main conclusion of this paper is that by itself an accuracy metric such as absolute relative error tells us very little about the *quality* of an embedding as experienced by a user.

Our own experiences and experiments with Internet coordinate systems have been disappointing in several respects. First, distance estimation results are often *unpredictable* in the sense that many nodes obtain good estimates while a few obtain extremely bad results. In a real-world setting it seems that nodes cannot determine the *quality* of their estimates without performing exactly the full probing that coordinate systems are intended to eliminate. Second, we observed that the *quality* of an embedding often varied considerably with small changes to the topology of the underlying network — something which would be beyond the control of most users of a coordinate system. Third, we observed that the *quality* of embeddings often varies considerably as the number of participating nodes changed, even when the underlying topology remains fixed. In short, these observations suggest that it is very hard to predict when a coordinate system will work well at any given node.

Highly aggregated accuracy metrics, such as absolute relative error, seem to give little indication of these types of *quality* problems. This experience led us to question the usefulness of metrics such as absolute relative error, at least in isolation from other means of assessing *quality*. This paper presents several new accuracy metrics that attempt to capture more application-centric notions of *quality*. The first is called *relative rank loss at node A* (*rrel* at node A), and is intended to be useful for applications used by nodes that need to know only their *relative* distance of other nodes. That is, node A needs to answer question such as “Which node is closer to me, B or C?” — which we will call A’s proximity query for nodes B and C. The metric *rrel* at node A is defined as a type of *swap distance* and takes values between 0 (all proximity queries at A are correctly estimated) and 1 (none of the proximity queries at A are correctly estimated). For example, an *rrel* at node A of 0.5 tells us that node A has a 50 percent chance of getting the

correct answer to a proximity query. We then define aggregate versions of rrl such as the average rrl over all nodes, and the maximal rrl value over all nodes. The other new metric we define is *closest neighbors loss at node A* (cnl at node A). This metric is intended to be useful to nodes using applications that are interested only in determining which nodes are *closest*. The value of cnl at node A is 0 if the nodes closest to A remain closest in the embedding, and 1 otherwise. Again, we then define aggregate versions of cnl such as the average global cnl over all nodes. For example, an average global cnl value of 0.5 would tell us that 50 percent of the nodes have a different set of closest neighbors in the original space than they have in the embedded space.

When we compare the accuracy of embedding techniques using our new metrics the results can be very poor, even in simple tree and hub-and-spoke networks. Our new metrics capture and quantify, at least partially, some of our disappointment with the *quality* of the embedding techniques. However, it remains that no single accuracy metric can capture the *quality* of an embedding technique. It seems better to develop a collection of accuracy metrics that are each designed to quantify a specific aspect of user-oriented *quality*. By no means do we want to suggest that our new metrics represent a complete or definitive set — they are simply an initial attempt at exploring this space.

This paper is complementary to the work presented in [23], which shows that RTT violations of the triangle inequality are a common, persistent, and widespread consequence of Internet routing policies. We strongly suspect that such violations will degrade *every* embedding technique with respect to *any* accuracy metric, at least when the embedding maps into a geometric space where the triangle inequality does hold. The results of the current paper and those of [23] are not very encouraging, to say the least. On the positive side, we feel that research is always improved with a better understanding of the fundamentals. In this case, we believe an improved understanding suggests a new and potentially interesting line of research, which we will call Embeddable Overlay Networks (EONs). An EON is an overlay where routing nodes have been *selected* to avoid violations of the triangle inequality (for overlay forwarding) *and* the overlay topology has been *selected* to embed with high accuracy with respect to multiple useful accuracy metrics. To do this well will require new insights and new algorithms.

Paper Outline. Section 2 presents a short survey of the embedding techniques described in the literature. Section 3 presents the basic terminology of network embeddings and our new accuracy metrics. In Section 4 we present an embedding experiment using our data collected on PlanetLab. We use a *full* Lipschitz embedding technique which makes use of all nodes as the landmarks (reference nodes) without any dimensionality reduction, and evaluate the results with our new metrics. In Section 5, we conducted exper-

iments using our PlanetLab data sets and new accuracy metrics, but on other techniques that are based on numerical minimization — Vivaldi [8] and Big Bang Simulation (BBS) [18–20]. In Section 6, we revisit these previously published experiments and use their data sets to re-evaluate their accuracy with our new accuracy metrics. Section 7 concludes with some topics for future work.

2 A Brief Survey of Embedding Techniques

Among the recently described Internet coordinate systems are the Global Network Positioning (GNP) [16], Lighthouses [17], Virtual Landmarks [21], Internet Coordinate System (ICS) [11], Matrix Factorization [15], Practical Internet Coordinates (PIC) [7], Vivaldi [8], Big Bang Simulation (BBS) in Euclidean space [18], and BBS in Hyperbolic space [19, 20]. Basically, Internet coordinate systems comprise of *two* categories of network embeddings of finite metric space into low dimensional Euclidean or non-Euclidean (e.g. Hyperbolic) space, namely, *numerical minimization* of some defined objective distance error functions for nodes-to-landmarks distances; and initial Lipschitz embedding with some form of dimensionality reduction using *distance matrix factorization* (which is a form of mathematical optimization and minimization techniques of error functions).

ICS and Virtual Landmarks systems are based on Lipschitz embedding of a finite metric space into Euclidean space and use the Principal Component Analysis (PCA) to reduce dimensionality. Typically, the accuracy of such embeddings is studied *only after* some type of dimensionality reduction technique has been applied. Matrix factorization based on Singular Value Decomposition (SVD) is of the form $D = USV^T$, where D is the distance matrix, U and V are orthogonal matrices and S is a diagonal matrix with nonnegative elements arranged in decreasing order which measure the significance of the contribution from each principal component. This is related to PCA on the distance matrix row vectors in ICS and Virtual Landmarks, where the first d rows of the matrix U are used as coordinates for the network nodes. The Matrix Factorization embedding uses the distance matrix whose rows are not linearly independent (has rank strictly less than n where n is the number of nodes). It is expressed as the product of two smaller matrices of A and B (through SVD or Nonnegative Matrix Factorization (NMF) algorithms), which contain the outgoing and incoming vectors respectively for each node. Estimated distances between nodes are derived from the dot product of these *two* vectors. Lighthouses technique makes use of multiple local bases L together with a transition matrix in vector space, to allow flexibility for any node to determine coordinates relative to any set of landmarks provided it maintains a transition matrix for global basis G . It uses linear matrix factorization such as the QR

decomposition (Gram-Schmidt orthogonalization).

For techniques involving minimization of some defined objective distance error functions, many algorithms are proposed to compute the coordinates of the network nodes. For example, GNP and PIC systems use the Simplex Downhill to minimize the objective distance error function: sum of relative errors. The problems of using the Simplex Downhill method is the slow convergence, sensitive to the *initial* coordinates or position of the network nodes, and the potential of getting stuck in the local minima. This leads to the eventual assignment of different coordinates for the same node depending on the minimization process (e.g. selection of the initial position). So, methods that are clever to converge to the global minimum are required. Both Vivaldi and BBS (Euclidean and Hyperbolic spaces) algorithms are based on the numerical minimization of sum of distance error functions that are related to the problem of minimizing the potential energy of newtonian mechanics principles. Vivaldi system is constantly adjusting the coordinates of the nodes because each node contacts a random set of nodes in a decentralized manner. Each node cannot find a global minimum fit, and it cannot guarantee that a given error function minimization does not increase the global error, although Vivaldi ensures nodes always decrease the local error. Hence, the error term in each small time-step is monitored to derive the optimal sets of coordinates selected.

The Big-Bang Simulation (BBS) (Euclidean and Hyperbolic) systems model the network nodes as a set of particles, having a position in Euclidean space. The nodes are traveling under the effect of the potential force field which reduces the potential energy of the nodes. This is related to the total embedding distance error of all node pairs in Euclidean space. Each node pair is effected by the field force induced between them depending on their node pair’s embedding distance error, i.e. the embedding distance error of the distance between the node pair. The node is also affected by simulated friction force. BBS systems have the advantage over conventional gradient minimization schemes, such as steepest descent and Simplex Downhill, due to the kinetic energy accumulated by the moving nodes which enable them to escape the local minima. Hyperbolic geometric space is the target embedding space for the BBS (Hyperbolic) system. A distance decreases as it moves away from the origin. Similar to Euclidean line distance, the Hyperbolic distance line between two nodes is defined as the parametric curve, connecting between the nodes, over which the integral of the arc length is minimized. Unlike Euclidean line distance, a Hyperbolic line distance bends towards the origin point. The extent to which the bend depends on the curvature of the Hyperbolic space. The bending becomes larger when space curvature increases, and thus, Hyperbolic distance between two nodes increases. There are three embedding methods in

BBS systems: **All Pairs (AP) embedding** — Embedding is done for *full* mesh of the n nodes of the network topology comprising of $\frac{n(n-1)}{2}$ distance pairs (n is the number of network nodes). **Two Phases (TP) embedding** — Embedding is done using landmarks similar to GNP, where the landmarks are embedded first and the coordinates of the other nodes are calculated from the distance to *several* chosen *closest* landmarks through minimization of the symmetric distortion in these node-to-landmark pairs. Specifically, TP embedding requires $k + 1$ landmarks’ measurements for k -dimensional coordinate vectors. **Log-Random and Neighbors (LRN) embedding** [20] — It aims to increase neighbor distances accuracy. The LRN embedding concurrently embed nodes through minimization of objective error function of n nodes and LRN subset, which comprises of the node pairs whose distance is below a certain threshold, i.e. the threshold is selected so that the number of distance pairs that are below the threshold is $O(n \cdot \log n)$, and together with a set of randomly sampled distance pairs that are selected uniformly at random with probability $\frac{\log n}{n}$. The number of randomly sampled distance pairs is equivalent to $n \cdot \log n$. The LRN algorithm embeds all the n nodes concurrently. The objective function is the sum of embedding errors for all n nodes in the system, and the embedding error of one node is the error of distance from that node to the LRN subset of nodes.

3 Embeddings and Their Accuracy

This section rigourously defines several accuracy metrics for embeddings. We use the Lipschitz embedding as a running example to illustrate and motivate our metric definitions.

A *metric space* is a pair $M = (X, d)$ where X is a set equipped with the distance function $d : X \rightarrow \mathbb{R}^+$ — for each $a, b \in X$ the *distance between a and b* is given by the function $d(a, b)$. We require that for all $a, b, c \in X$,

(anti-reflexivity) $d(a, b) = 0$ if and only if $a = b$,

(symmetry) $d(a, b) = d(b, a)$,

(triangle inequality) $d(a, b) \leq d(a, c) + d(c, b)$.

Note that we will ignore the fact that in reality, violations of the triangle inequality exist for Internet RTTs ([23]).

Suppose that $M_1 = (X_1, d_1)$ and $M_2 = (X_2, d_2)$ are metric spaces. Every one-to-one function ϕ from X_1 to X_2 naturally defines a metric space called *the embedding of M_1 in M_2 under ϕ* , defined as $\phi(M_1) = (\phi(X_1), d_2)$. We normally abuse terminology and simply say that ϕ embeds X_1 into X_2 , leaving the distance functions to be inferred from the context. If ϕ embeds X_1 in X_2 and Z is a subset of X_1 , then ϕ also defines an embedding of Z in X_2 in the natural way. One simple case is where X_1 is a finite set,

X_2 is \mathbb{R}^n with the standard notion of Euclidean distance, $d_2(x, y) = \|x - y\| = \sqrt{\sum_{1 \leq i \leq n} (x_i - y_i)^2}$.

If $L = \{l_1, l_2, \dots, l_m\} \subseteq X$, then we can map each $a \in X$ to a point in \mathbb{R}^m as $\phi_L(a) = (d(a, l_1), d(a, l_2), \dots, d(a, l_m))$. This is called the *Lipschitz embedding of X using landmarks L* . If $L = X$, we refer to $\phi_X = \phi$ as the *full Lipschitz embedding*.

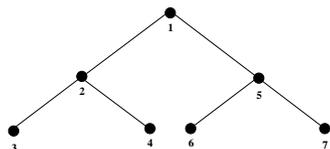


Figure 1: A binary tree of depth 2.

We illustrate the *full* Lipschitz embedding with a simple example. Any weighted undirected graph $G = (V, E, w)$ naturally gives rise to a metric space $M(G) = (V, d)$, where $d(a, b)$ is the length of a shortest path between nodes a and b . Figure 1 presents one example — a simple binary tree of depth 2, where each edge represents a distance of 1, and gives rise to the following distance matrix:

	1	2	3	4	5	6	7
1	0	1	2	2	1	2	2
2	1	0	1	1	2	3	3
3	2	1	0	2	3	4	4
4	2	1	2	0	3	4	4
5	1	2	3	3	0	1	1
6	2	3	4	4	1	0	2
7	2	3	4	4	1	2	0

Note that this distance matrix itself can be viewed as a full Lipschitz embedding into a \mathbb{R}^7 by reading each row as the 7-dimensional coordinate of the associated node. For example, $(0, 1, 2, 2, 1, 2, 2)$ are the coordinates of node 1.

3.1 Relative Error

We seek embeddings that are *accurate*. There are several ways to capture this formally, and in the context of Internet coordinate systems the most appropriate notion may depend on the needs of an application. Some applications require that the distances in the embedding accurately reflect the original distances. Note that with the binary tree example above distances are not well preserved — we can calculate the distance between $\phi(1)$ and $\phi(7) = (2, 3, 4, 4, 1, 2, 0)$ as approximately 4.47, yet it is only 2 in the original metric space.

If we multiply each coordinate $\phi(x)$ by a scalar value β , and redefine the embedding as $\phi'(x) = \beta\phi(x)$, we could improve this notion of accuracy. As in [21], we choose β to minimize the *absolute relative error* for all pairs of nodes, given by $\sum_{x, y \in X} \frac{\|\phi'(x) - \phi'(y)\| - d(x, y)}{d(x, y)}$. For the

example above, this gives a value for β of approximately 0.5, which reduces the distance between the embeddings of nodes 1 and 7 to approximately 2.3. The *average absolute relative error* is obtained by dividing the absolute relative error with $n(n - 1)$, where n is the total number of nodes. The *maximum local absolute relative error* is the maximal *absolute relative error* over all nodes.

A similar accuracy metric is *stress*, defined as $\frac{\sum_{x \neq y \in X} (\|\phi'(x) - \phi'(y)\| - d(x, y))^2}{\sum_{x \neq y \in X} d(x, y)^2}$. Note that both stress and relative error quantify the magnitude of the differences between original distances and embedded distances. If stress (or relative error) is 0, then we have an embedding that preserves distances exactly (an isometry).

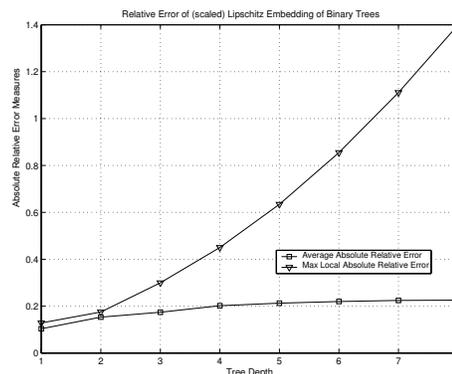


Figure 2: Absolute Relative Error measures for Lipschitz embedding on binary trees, depth 1 to 8.

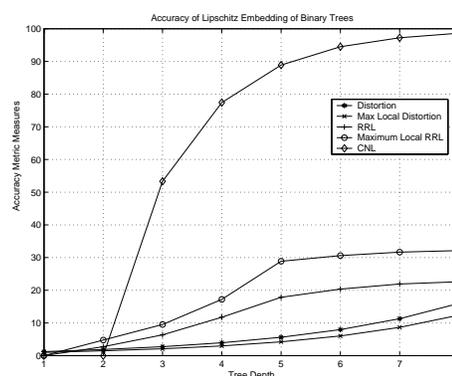


Figure 3: Scalar independent measures for Lipschitz embedding on binary trees, depth 1 to 8.

3.2 Distortion

In the theoretical literature on embeddings (for example, [10, 12–14]) the most common notion of accuracy is captured by *distortion*. This measure is invariant under scalar

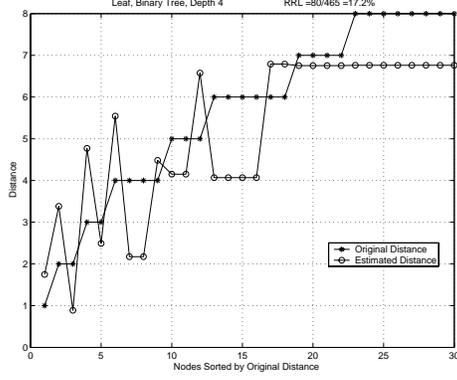


Figure 4: The view from a leaf in a binary tree of depth 4.

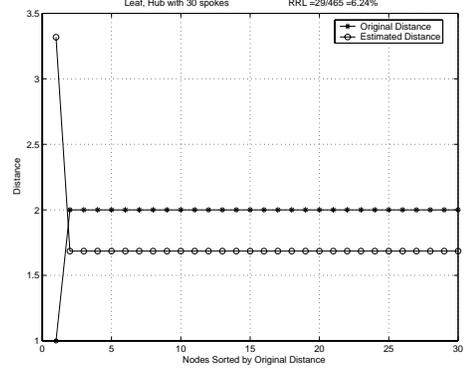


Figure 6: The view from a leaf in a hub with 30 spokes.

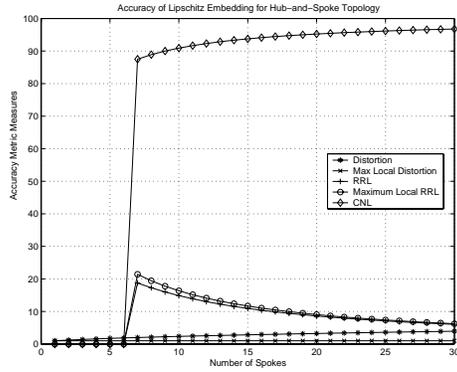


Figure 5: Hub and Spoke accuracy.

multiplication. That is, $distortion(\phi) = distortion(\alpha\phi)$, for all $\alpha \neq 0$ — and so provides a notion of accuracy that is independent of the absolute values of distances. Distortion measures the worst-case change in the *relative distances* of the embedding.

First, define the ratio $r(\phi, x, y) = \frac{\|\phi(x) - \phi(y)\|}{d(x, y)}$. The *expansion* of ϕ , $expansion(\phi)$, is the maximum value of $r(\phi, x, y)$ as x and y range over X ($x \neq y$). The *contraction* of ϕ , $contraction(\phi)$, is the minimum value of $r(\phi, x, y)$ as x and y range over X ($x \neq y$). The *distortion* of ϕ is defined as the ratio $distortion(\phi) = \frac{expansion(\phi)}{contraction(\phi)}$. Note that the $distortion(\phi)$ is always greater than or equal to 1. The equality holds only when there exists a constant α such that $r(\phi, x, y) = \alpha$ for every x, y . In this case $\phi'(x) = \phi(x)/\alpha$ is an isometry.

One way to gain intuition for the meaning of distortion is to imagine that $contraction(\phi) = 1$ (which could always be achieved with scalar multiplication). In this case there is no contraction in the embedding, only expansion, and the largest expansion is achieved by some pair x, y , where the distance between x and y in the embedding is $expansion(\phi)$ times the original distance $d(x, y)$.

Note that distortion is a global worst-case measure, which may be relevant to applications (such as mapping) that require a global picture of the entire embedding. However, most Internet coordinate applications will be interested only in *local distortion* — the amount of distortion seen from any one node. For a fixed x , we define $expansion(\phi, x)$, to be the maximum value of $r(\phi, x, y)$ as y range over X ($x \neq y$), and $contraction(\phi, x)$, to be the minimum value of $r(\phi, x, y)$ as y range over X ($x \neq y$). Then the local distortion is defined to be $distortion(\phi, x) = \frac{expansion(\phi, x)}{contraction(\phi, x)}$. The *maximum local distortion* is the maximal $distortion(\phi, x)$ over all x .

3.3 A New Metric: Relative Rank Loss (*rrel*)

Many applications only need to know the *relative* distance of other nodes. They need to answer question such as "Is A closer than B?" What is important for such applications is that the *relative rankings of distances is not lost*. For this notion of accuracy we define *relative rank loss (rrel)* to be between 0 (for no loss of relative order) and 1 (for a complete reversal of order). This is a slight generalization of *swap distance*, a well-known edit distance on strings. For example, between strings $s_1 = abcd$ and $s_2 = acdb$ there is a swap distance of 2. We need to generalize this slightly to account for the fact that multiple nodes can be equidistant from a given node.

First, define the function

$$R(x, y) = \begin{cases} -1 & \text{if } x < y \\ 0 & \text{if } x = y \\ 1 & \text{if } x > y \end{cases}$$

We say that x and y are swapped w.r.t. z if $R(d(z, x), d(z, y)) \neq R(\|\phi(z) - \phi(x)\|, \|\phi(z) - \phi(y)\|)$ and denote this as $swapped(z, x, y)$. That is, if $swapped(z, x, y)$ holds, then z 's relative distance relationship to x and y is different in the original and embedded

spaces. Define $P(z)$, a subset of $(X - \{z\}) \times (X - \{z\})$, to be

$$\{(x, y) \mid x \neq y \text{ and } \text{swapped}(z, x, y)\}.$$

Finally, define the *local relative rank loss at z* to be

$$\text{rrl}(\phi, z) = \frac{|P(z)|}{s},$$

where $s = \frac{(|X|-1)(|X|-2)}{2}$. Note that $\text{rrl}(\phi, z)$ is between 0 and 1. It can be interpreted as the probability that the relationship between any two nodes in $(X - \{z\}) \times (X - \{z\})$ will have a different relative order (from z 's point of view) in the original and embedded spaces (assuming all pairs (x, y) have equal probability of being chosen).

Define the *maximal local relative rank loss at z* to be the maximal value of $\text{rrl}(\phi, z)$ as z ranges over X . The *average local relative rank loss at z*, denoted $\text{rrl}(\phi)$, is defined as $\frac{\sum_{x \in X} \text{rrl}(\phi, x)}{|X|}$.

3.4 A New Metric: Closest Neighbors Loss (*cnl*)

Some applications are interested only in determining which nodes are *closest*, and so require that embeddings accurately preserve the set of closest nodes. For a node x , we define its *local closest neighbors loss* to be 0 if the nodes closest to x in X are mapped to the nodes closest to $\phi(x)$, and 1 otherwise. We denote this value by $\text{cnl}(\phi, x)$. If we take the average over all nodes in X , we denote the global value as $\text{cnl}(\phi)$. We often multiply this global *cnl* by 100 and speak of the percentage of nodes whose nearest neighbors are not preserved.

3.5 Examples of Inherent Inaccuracy — Topology Matters

We now apply the accuracy measures defined above to several simple topologies. Figure 2 presents the relative error and maximum local relative error measures for binary trees of depth 1 (3 nodes) through 8 (511 nodes). Note that it is not obvious or intuitive how to interpret these (small) values. On the other hand, Figure 3 presents the scalar-independent measures for the same set of binary trees. The plot for *cnl* tells us that about 96 percent of the 511 nodes in a tree of depth 8 have a different closest neighbors set in the Lipschitz embedding. The plot for *rrl* shows that on average nodes see over 20 percent of their relative distance relationships swapped. The plot for maximal local *rrl* tells us that at least one node saw over 30 percent of its relative distance relationships swapped.

Figure 4 helps us to understand these inaccuracies. This plot is from the perspective of a leaf node in a binary tree of depth 4 (31 nodes). The signature plot marked with *'s

shows the distances of other nodes from this leaf, in ascending order. The signature plot marked with **o**'s shows the distances of the same nodes in the Lipschitz embedding. In the original metric space, the parent of this leaf is the only node at distance 1. But in the embedding, this node moves to a distance of about 1.8. There were two nodes originally at distance 2 — the leaf's sibling and its parent's parent. One of these (its sibling) has moved closer (and is now closer than the leaf's parent), to a distance just under 1, while the leaf's parent's parent moves out to a distance of about 3.4. The local *rrl* for this node is 17.2 percent. Note that it has lost its closest neighbor in the embedding.

Some topologies show a very different relationship between *rrl* and *cnl*. For example, Figure 5 shows the scalar-independent measures for a family of hub-and-spoke networks. The graph have n spokes and 1 root, where n ranges from 1 to 30. Here we see a rising *cnl* and a falling *rrl* after $n = 6$. The reason becomes clear when we look at the view from a leaf node, as shown in Figure 6. We see that the root nodes is *pushed away* to a distance of about 3.3.

3.6 The Scalability (Meta-) Metric

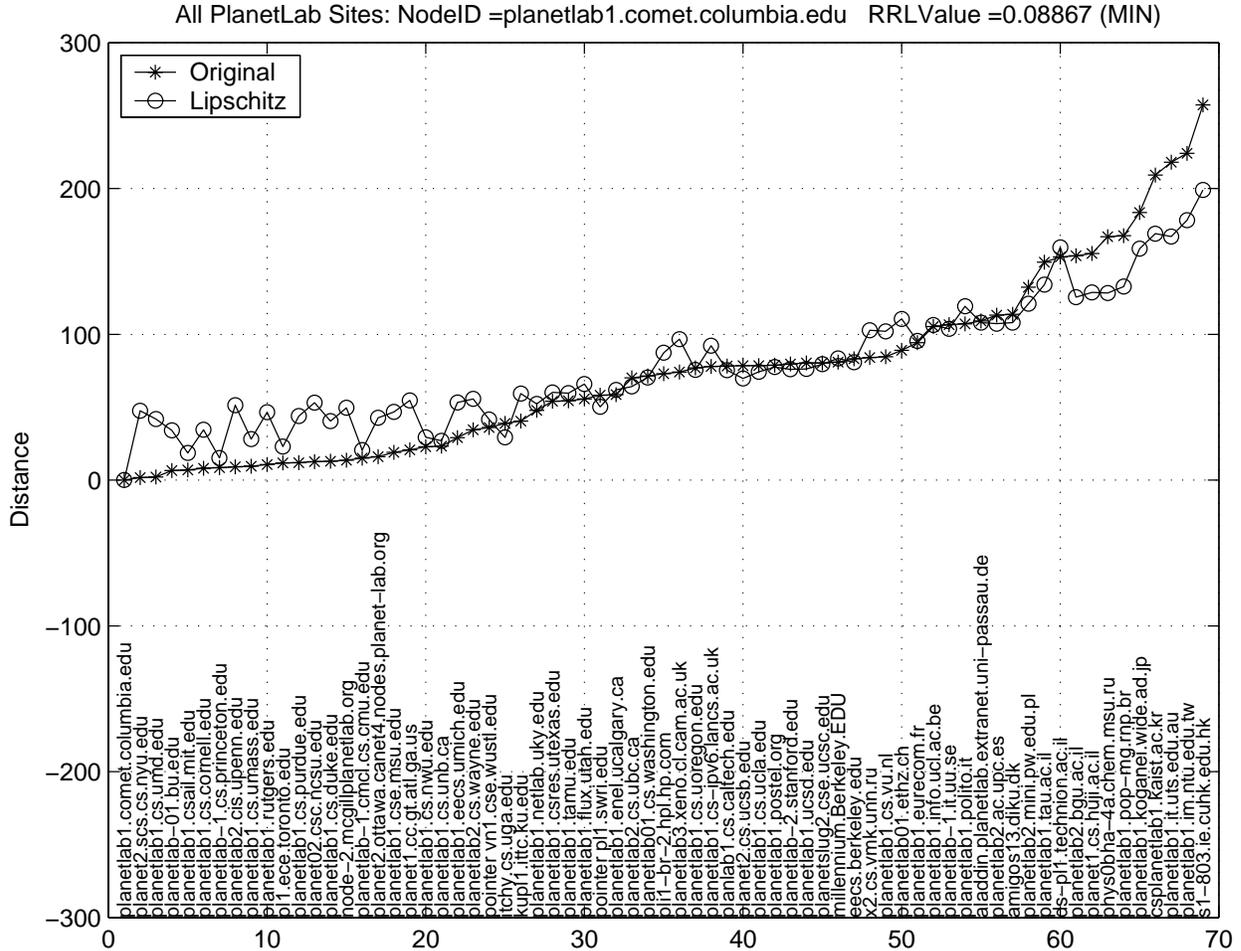
Suppose that a set of applications is only interested in a subset of nodes, say just the North American sites. Would it be better to use a coordinate system generated for all sites, or one generated just from inter-site RTTs restricted to North American sites? The answer to such questions will determine if embeddings of coordinate *services* could be *truly* scalable.

If Y is a subset of X , we can obtain an embedding in one of two ways. First, we could use the embedding techniques to obtain $\phi(X)$, and then restrict this to the nodes in Y , which we will denote as $\phi(X) \downarrow Y$. This is called *Superspace* embedding. Alternatively, we could use the embedding techniques on Y to get $\phi(Y)$, which we call the *Subspace* embedding. In general, $\phi(X) \downarrow Y$ and $\phi(Y)$ may be *very* different embeddings and have *very* different behaviors with different accuracy measures.

For example, consider again Figure 3. Let X be the binary tree of depth 8. We can consider each of the subtrees of depths 1 to 7 as a Subspace Y of X . The figure shows the scalar-independent accuracy measures for each full Lipschitz embedding of Subspace $\phi(Y)$. However, the full Lipschitz embeddings derived from the Superspace, $\phi(X) \downarrow Y$, will in general inherit some of the inaccuracies of the Superspace embedding — that is the nodes outside of Y in X impact the way Y is embedded.

4 An Experiment with the Lipschitz Embedding

Although data sets studied in previous research comprised large number of measurements, some of them were not



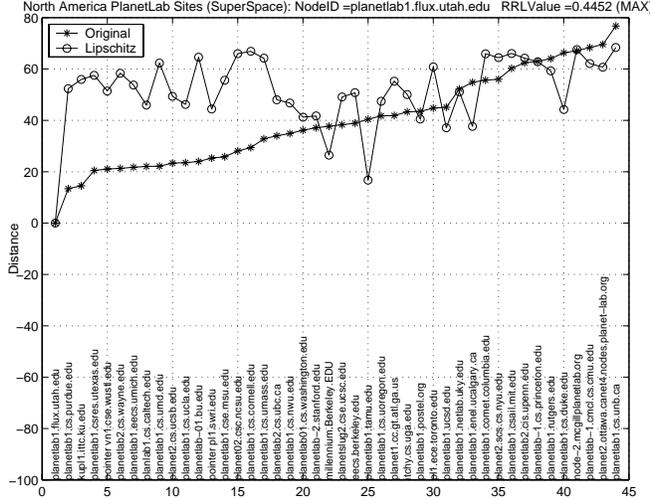
full meshes. The Skitter project [3] used in Virtual Landmarks [21], for instance, makes RTT data available from a small number of monitoring nodes n to m target nodes, where m is in the order of hundreds of thousands. This yields an asymmetric data set of $n \times m$ where only estimated distances between monitoring nodes and target nodes can be verified; distances between target nodes cannot be determined. This issue can be overcome with mesh data sets. We acknowledge the fact that finding representative mesh data sets is not an easy task. However, planetary-scale testbeds such as PlanetLab [2] provides a distributed platform not only for overlay-based services but also for network measurements.

We used RTT data¹ collected between all PlanetLab nodes from March 22nd to March 28th 2004. During this time period, there were over 150 participating sites with a total of over 370 hosted nodes distributed in the Americas, Europe, Asia and Australia. We then calculated the minimum RTT between each pair of nodes available between those dates on consecutive 15-minute periods. Thus,

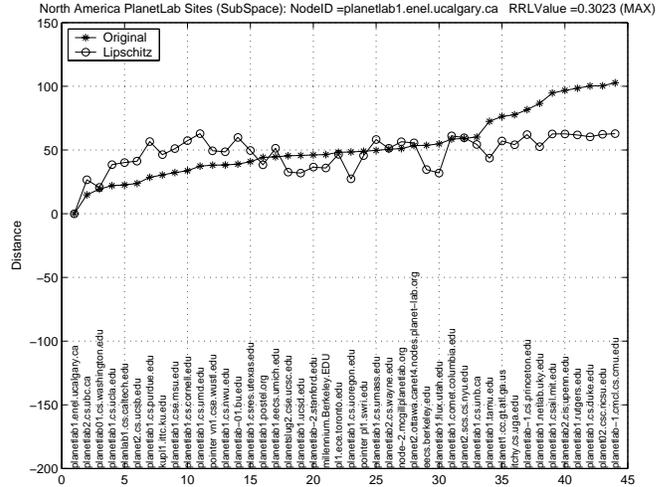
for each day in this period there were 96 matrices of RTT measurements, and the size of each matrix was 325×325 . Therefore, over the seven days period, we obtained 672 such matrices.

Since we wanted to perform this analysis only over our estimate of propagation delays on the paths (and not the queuing and congestion effects), we first constructed a single distance matrix D , in which each entry represented the minimum RTT between a pair of PlanetLab nodes over the entire 7-day period. This avoided biases in the results due to high variations in RTTs, e.g. during congested periods. Our analysis of the data indicated that by taking the minimum over a 7-day period, we can filter out congestion related effects which have periodic weekly patterns.

Many PlanetLab sites had multiple nodes per site. For instance, the Computer Laboratory (University of Cambridge) site hosted three nodes planetlab1, planetlab2 and planetlab3 under the domain cl.cam.ac.uk. The minimum RTTs between nodes within a site were very small, often of the order of 1 ms.



(a) North America (*Lipschitz* Superspace embedding). PlanetLab site with maximum *rrl*.



(b) North America (*Lipschitz* Subspace embedding). PlanetLab site with maximum *rrl*.

Figure 9: PlanetLab site with maximum *rrl* using full *Lipschitz* Superspace and Subspace embeddings.

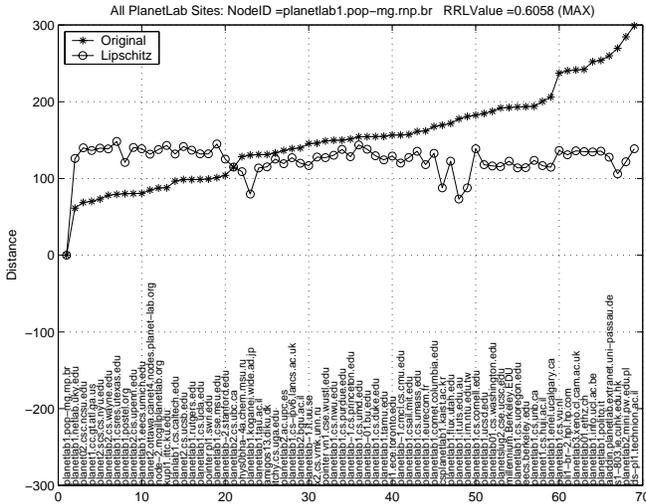


Figure 8: PlanetLab site `planetlab1-pop-mg.mp.br` with maximum *rrl* using *full* Lipschitz embedding.

Examining RTTs between all pairs of nodes was therefore wasteful and biased our results by the distribution of nodes in PlanetLab sites and by the fact that there were missing entries on the original 325×325 distance matrix. Thus, we selected a representative node in each site so as to build a *site by site* distance matrix D' , and then we removed rows and columns that contained missing entries. This process reduced the distance matrix to 69×69 RTTs between sites. Finally, we used the classification proposed in [5] to split D' into three sub-matrices of different sizes based on the geographical locations of PlanetLab sites:

North America (NA-PL): 44×44 distance matrix

which contains RTTs between PlanetLab sites located in North America. The majority of these sites obtain interconnectivity through the Abilene network.

Outside North America (ONA-PL): 25×25 distance matrix with RTTs between *research* and *commercial* sites outside North America. It includes sites in Australia, Europe, Latin America and Asia.

All sites (ALL-PL): the combination of NA-PL and ONA-PL data sets, of size 69×69 .

We apply the *full* Lipschitz embedding to the RTT distance matrix of the 69 PlanetLab sites in **ALL-PL**. Here are the minimum, mean, and maximum values we found for local *rrl*:

	Min	Mean	Max
<i>rrl</i>	0.0887	0.1822	0.6058

The difference between the maximum and minimum *rrls* is high (51.71%). Figure 7 presents data from the PlanetLab site `comet.columbia.edu`, which had the minimum local *rrl* measure. The *x*-axis enumerates sites and the *y*-axis shows the RTT distance of each site from `comet.columbia.edu` (in milliseconds). The signature plot marked with *'s indicates RTTs in the original distance matrix, and the sites on the *x*-axis have been sorted to ensure that this plot is in *ascending* order of RTTs. The signature plot marked with o's is the geometric distances between the same sites in the embedded geometric space. For these signature plots, we have multiplied the Lipschitz embedding by a scaling factor that minimizes relative error (as described in Section 3). Of course, this does not impact the *rrl* or *cnl* measures, but it helps us visualize the differences between distances in the original and embedded spaces. Note that even in this best case (at least

with respect to local rrl), we see that the relative ranking of many close nodes is **reversed** and the list of close neighbors is **not preserved**. We saw similar results for binary trees (Section 3), and we suspect that many of the inaccuracies visible in Figure 7 may be a consequence of the inaccuracies inherent in the Lipschitz embedding for the underlying network topology. Note that a user of this embedding at `comet.columbia.edu` would conclude that `nyu.edu` was about as far away as `utexas.edu`!

Table 1: Local rrl of Lipschitz Subspace and Superspace embeddings using NA-ALL and ALL-PL PlanetLab sites.

Embeddings	Min rrl	Mean rrl	Max rrl
$\phi(\text{NA-PL})$	0.1141	0.1897	0.3023
$\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$	0.1606	0.2916	0.4452

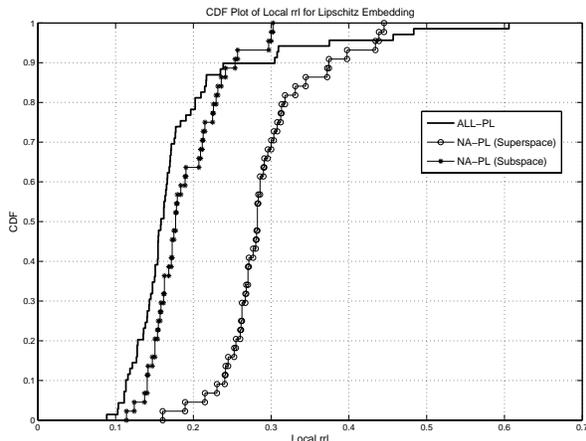


Figure 10: CDFs of Local rrl for Lipschitz Subspace and Superspace embeddings in Euclidean Space.

Figure 8 presents a signature plot for the site with the maximum local rrl measure, where the results here are not very impressive — site `pop-mg.rnp.br` has a 60.6% chance of swapping the relative distance of any two nodes. It would appear from looking at this plot that using this embedding at site `pop-mg.rnp.br` would give very poor results. Note that in these figures, the site has **lost** its list of neighbors’ rankings in the embedding. In fact, the global cnl measure is 84.06%, for ALL-PL data set, so only about 15% of the sites retain their closest neighbors in the embedding.

We proceed to explore scalability (meta-) metric of the Lipschitz embedding with our PlanetLab site data. We used the data sets NA-PL (North America) as a Subspace of the data set ALL-PL (All sites) of size 69. We call $\phi(\text{NA-PL})$ the **Subspace** embedding and $\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$ the **Superspace** embedding of NA-PL. The minimum, mean, and maximum local rrl for the full Lipschitz Superspace and Subspace embeddings are shown in Table 1. This is

visualized in Figure 9(a) and Figure 9(b), which show the nodes with the maximum $rrls$ for Lipschitz Superspace and Subspace embeddings. Note that the Lipschitz Subspace embedding in Euclidean space, is a **much better** one. Figure 10 shows the CDFs of the local rrl , for the Lipschitz $\phi(\text{ALL-PL})$, $\phi(\text{NA-PL})$ (Subspace), and $\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$ (Superspace) embeddings. It clearly shows that the Lipschitz Subspace embedding in **Euclidean space** is **better** than the Lipschitz Superspace embedding, at least with respect to the local rrl measure of accuracy.

5 Using Other Embeddings

We run experiments to apply our new accuracy metrics using Vivaldi and BBS (Euclidean and Hyperbolic) systems and our PlanetLab sites data. The **ALL-PL** (All sites) data set of 69 sites consisting of North America and Outside North America PlanetLab sites, are selected for this experiments. We utilize the *p2psim* simulator [1] (which Vivaldi embedding system is incorporated in), BBS (Euclidean) and BBS (Hyperbolic) TP and LRN embeddings systems, to generate 3-dimensional coordinates for our PlanetLab **ALL-PL** (All sites) data set. We utilize the TP embedding’s parameters of $t = 15$ landmarks (the first 15 nodes in the data set chosen as landmarks) and 15 node-to-landmark measurements (each of the other nodes measure their distance to 15 landmarks for distortion minimization). Table 2 shows the maximum, mean and minimum rrl and cnl accuracy metrics for Vivaldi and BBS (Euclidean and Hyperbolic spaces) embeddings using **ALL-PL** PlanetLab sites data.

Table 2: rrl and cnl accuracy metrics for Vivaldi and BBS (Euclidean and Hyperbolic) embeddings using **ALL-PL PlanetLab sites**

Embeddings	Min rrl	Mean rrl	Max rrl	cnl (%)
Vivaldi	0.0817	0.1476	0.3494	75.36
BBS (Euclidean)	0.0768	0.1263	0.2291	75.36
BBS TP (Hyperbolic)	0.0382	0.2391	0.6286	72.46
BBS LRN (Hyperbolic)	0.0680	0.1419	0.3670	68.12

Both BBS (Euclidean) and Vivaldi embeddings in Euclidean space have the **same** cnl measure of 75.36%, but Vivaldi embedding has a **higher** maximum rrl compared to BBS (Euclidean). The BBS (Hyperbolic) TP embedding has a **much higher** maximum rrl than BBS (Hyperbolic) LRN embedding, but its minimum rrl is **lower** than BBS (Hyperbolic) LRN embedding. BBS (Hyperbolic) LRN embedding has the **lowest** cnl measure of 68.12%, compared to the other embeddings. On the other hand, BBS (Euclidean) embedding has the **lowest** maximum rrl and BBS (Hyperbolic) TP embedding has the **largest** maximum

rrl compared to the other embeddings. Figures 11 and 12 show the signature plots of the PlanetLab node with maximum rrl for BBS (Hyperbolic) TP embedding in Hyperbolic space and Vivaldi embedding in Euclidean space respectively. Both graphs show their lists of close neighbors are being **pushed away** in the embedded target geometric space.

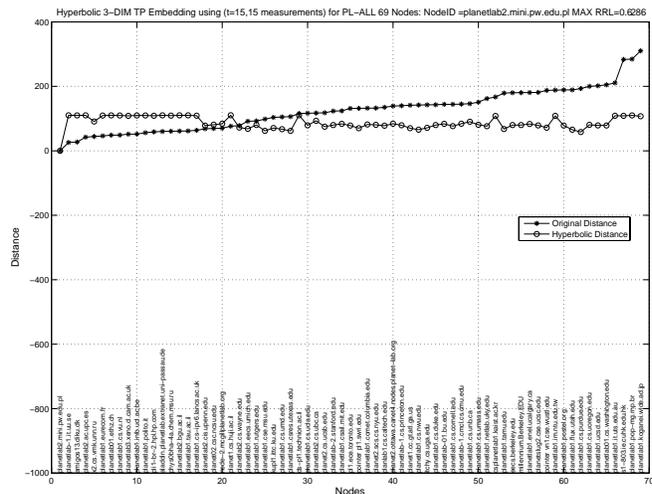


Figure 11: BBS (Hyperbolic) TP embedding using 69 PlanetLab **ALL-PL** (All sites) network topology — Nodes with Maximum rrl .

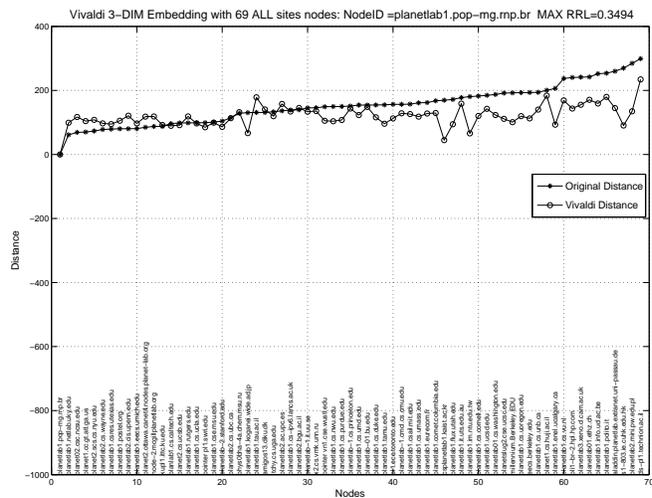


Figure 12: Vivaldi embedding using 69 PlanetLab **ALL-PL** (All sites) network topology — Nodes with Maximum rrl .

Similarly, we explore scalability (meta-) metric of the Vivaldi and BBS embeddings with our PlanetLab site data. The *Vivaldi* and *BBS (Euclidean)* Subspace and Superspace embeddings have the same behavior as the *Lipschitz* Subspace and Superspace embeddings, i.e. the Subspace em-

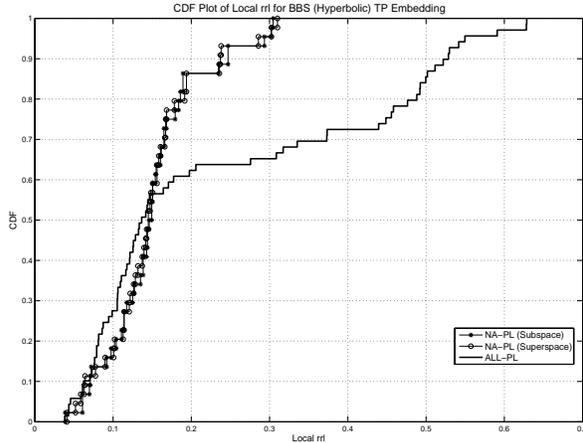
Table 3: Local rrl for *Vivaldi* and *BBS* Subspace and Superspace embeddings using NA-ALL and ALL-PL PlanetLab sites

Embeddings	Min rrl	Mean rrl	Max rrl
Vivaldi			
$\phi(\text{NA-PL})$	0.0853	0.1542	0.2625
$\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$	0.1351	0.23	0.4507
BBS (Euclidean)			
$\phi(\text{NA-PL})$	0.0819	0.1407	0.2614
$\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$	0.1440	0.2170	0.3544
BBS (Hyperbolic) TP			
$\phi(\text{NA-PL})$	0.0388	0.1512	0.3045
$\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$	0.0410	0.1496	0.3101
BBS (Hyperbolic) LRN			
$\phi(\text{NA-PL})$	0.0864	0.1713	0.3167
$\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$	0.0930	0.1504	0.2824

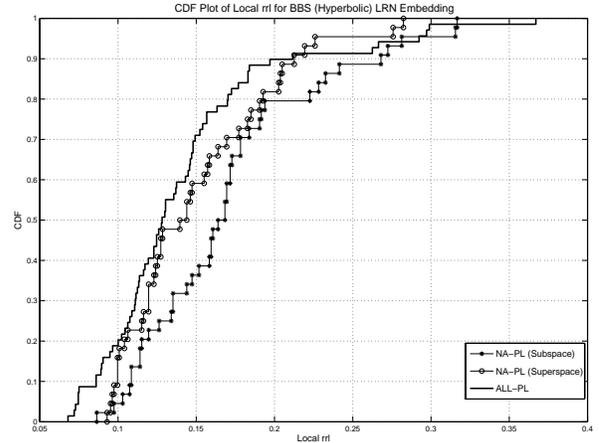
bedding has **better** rrl accuracy than the Superspace embedding in the **Euclidean space**. However, in *BBS (Hyperbolic) TP* and *BBS (Hyperbolic) LRN* Subspace and Superspace embeddings, the Subspace embedding rrl accuracy is **close** to the Superspace embedding as shown in the Table 3. For BBS (Hyperbolic) TP embedding using the NA-PL site data set, we adopt the same parameters as in the TP embedding for the ALL-PL, which is $t = 15$ landmarks (the first 15 nodes in the data set chosen as the landmarks), and 15 node-to-landmark measurements (each of the other nodes measure their distance to 15 landmarks for distortion minimization). Figure 13 shows the CDFs of the local rrl for the *BBS (Hyperbolic) TP* and *LRN* $\phi(\text{ALL-PL})$, $\phi(\text{NA-PL})$ (Subspace) and $\phi(\text{NA-ALL}) \downarrow \text{NA-PL}$ (Superspace) embeddings. This results suggest that the Superspace embedding tends to have a **close** or **better** rrl accuracy than the Subspace embedding in the **Hyperbolic space**.

6 Revisiting Previous Work with New Accuracy Metrics

It will be interesting to apply our new accuracy metrics to Vivaldi and BBS (Euclidean and Hyperbolic) systems (based on numerical minimization techniques) on *their* data sets that were used in their work. We generate 3-dimensional coordinates for these embeddings in our experiments. With reference to BBS systems [18–20], we utilize network topologies from the TRACER placement method (the network topologies chosen by selecting the first n nodes in the *whole* network) for our experiments due to its suitability for geometric distance experiments. We utilize the TP embedding’s parameters of $t = 10$ landmarks and 6 node-to-landmark measurements: this means that there are 10 landmarks chosen from the first 10 nodes in the network topology and *only* 6 *closest* measurements are carried out by other nodes to these 10 landmarks.



(a) CDFs of Local rrl for *BBS (Hyperbolic) TP* Subspace and Superspace embeddings.



(b) CDFs of Local rrl for *BBS (Hyperbolic) LRN* Subspace and Superspace embeddings.

Figure 13: CDFs of Local rrl metrics for Subspace and Superspace embeddings in Hyperbolic Space.

Table 4: rrl accuracy metric for Vivaldi and BBS (Euclidean and Hyperbolic) embeddings

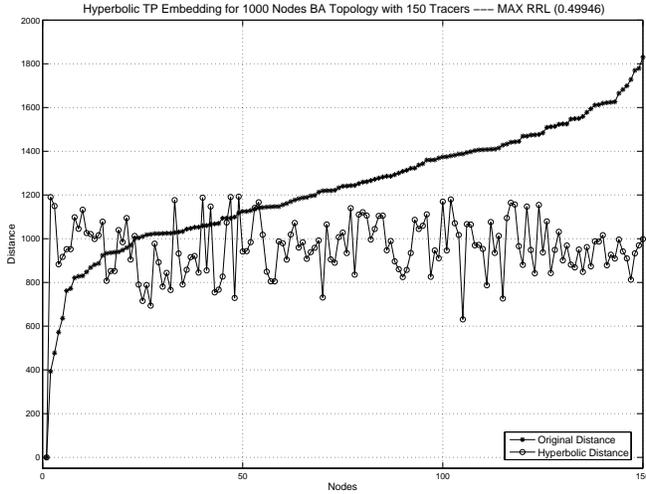
Embeddings	Data sets	Min rrl	Mean rrl	Max rrl
Vivaldi	192 PlanetLab filtered nodes	0.0334	0.0894	0.2808
Vivaldi	1740 King nodes	0.0633	0.1306	0.5667
BBS (Euclidean)	Topology of 50 nodes chosen among 600 nodes in Waxman Network	0.1310	0.2090	0.3639
BBS (Euclidean)	Topology of 15 nodes chosen among 1000 nodes in BA Network	0.0220	0.0982	0.2857
BBS (Euclidean)	Topology of 150 nodes chosen among 6474 nodes in Jan 2000 AS Network	0.3531	0.4553	0.6379
BBS (Hyperbolic) TP with 10 landmarks and 6 measurements	Topology of 150 nodes chosen among 1000 nodes in BA Network	0.0892	0.2452	0.4995
BBS (Hyperbolic) TP with 10 landmarks and 6 measurements	Topology of 150 nodes chosen among 6474 nodes in Jan 2000 AS Network	0.3305	0.4636	0.6452
BBS (Hyperbolic) TP with 10 landmarks and 6 measurements	Topology of 50 nodes chosen among 600 nodes in Waxman Network	0.1573	0.2647	0.4524
BBS (Hyperbolic) TP with 15 landmarks and 15 measurements	Topology of 200 nodes with <i>lower</i> degree chosen among 10670 nodes in Mar 2001 AS Network	0.0228	0.1419	0.2153
BBS (Hyperbolic) LRN	Topology of 200 nodes with <i>lower</i> degree chosen among 10670 nodes in Mar 2001 AS Network	0.0802	0.1796	0.2389

There are *two* data sets used in the Vivaldi [8] work. **PlanetLab RTT data of filtered nodes:** This is pair-wise minimum RTTs of 192 PlanetLab nodes over a certain period of time (not specified in [8]), from the PlanetLab "ping" collection traces. It is filtered due to some nodes in PlanetLab having security measures restricting "ping" connection from other nodes. **King [9] data:** This data set contains 1740 Internet DNS servers RTT traces based on the King collection method.

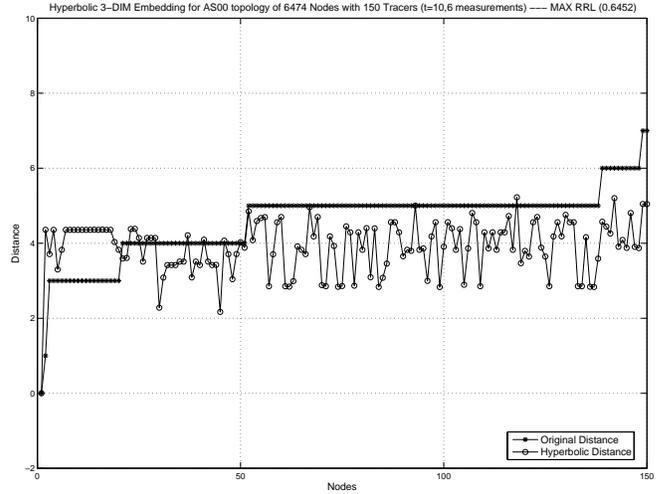
We selected the following data sets from the BBS (Euclidean) [18] work. **Waxman [22] Network Topology:** This data set contains undirected fixed network topologies of 600 nodes and the edge weights are generated with random uniform distribution in the range of $[1, 1000]$. The edge weights are taken as $\lfloor 10^E + 0.5 \rfloor$, where E is randomly and uniformly distributed in the interval $(0, 3]$. Based on the 600 nodes' connectivity, a Dijkstra algorithm is performed to find the shortest path distance matrix.

Network topologies consist of the first 50 nodes chosen among the 600 nodes in the Waxman Network. **Barabási-Albert [4, 6] (BA) Network Topology (considered to follow the PowerLaw):** This data set consists of undirected fixed network topologies of 1000 network nodes and the edge weights are generated with random uniform distribution in the interval of $[1, 1000]$. Based on the 1000 nodes' connectivity, a Dijkstra algorithm is performed to find the shortest path distance matrix. Network topologies consist of the first 15 nodes chosen among the 1000 nodes in the BA Network.

We used the following data sets from BBS (Hyperbolic) [19, 20] work. **Barabási-Albert (BA) [4, 6] Network Topology (considered to follow the PowerLaw):** This data set consists of undirected fixed network topologies of 1000 nodes, and edge weights are generated with random uniform distribution in the interval of $[1, 1000]$ (same data set in BBS (Hyperbolic) work [19]). Network topologies



(a) Node with Maximum rrl using network topology of 150 nodes chosen from 1000 nodes in BA Network Topology.



(b) Node with Maximum rrl in network topology of 150 nodes chosen from 6474 nodes in Jan 2000 AS Hierarchical Tree Network Topology.

Figure 14: BBS (Hyperbolic) TP embedding with 10 landmarks and 6 measurements — Nodes with Maximum rrl .

consist of the first 150 nodes chosen among the 1000 nodes. A Dijkstra algorithm is performed to find the shortest path distance matrix. **January 2000 AS Network Topology from University of Oregon (RouteViews):** This hierarchical tree network topology consists of 6474 nodes, and edge weights are generated with random uniform distribution in the interval of $[1, 1000]$ (same data set in BBS (Hyperbolic) work [19]). Network topologies consist of the first 150 nodes chosen among the 6474 nodes. A Dijkstra algorithm is performed to find the shortest path distance matrix. **March 2001 AS Network Topology from University of Oregon (RouteViews):** This data set is only used in the BBS (Hyperbolic) LRN embedding work [20]. It consists of a hierarchical tree network topology of 10670 nodes that are randomly weighted, with edge weights distributed uniformly in the interval of $[1, 1000]$. We extracted 200 weighted network topology of nodes with lower degree, or stub ASes.

Table 4 summarizes the maximum, mean and minimum rrl accuracy metric for Vivaldi and BBS (Euclidean and Hyperbolic) embeddings using their data sets [8, 18–20]. The signature plots of the nodes having maximum rrl for BBS (Hyperbolic) TP embedding in Hyperbolic space are shown in Figure 14. Evidently, the result of TP embedding in Hyperbolic space using AS Hierarchical Tree Network Topology shows **similar** rrl inaccuracy behavior as the Lipschitz embedding in Euclidean space from the perspective of the tree node in the synthetic binary trees which we have illustrated in Section 3.5. This confirms that BBS (Hyperbolic) TP embedding in **Hyperbolic space** has the **similar** rrl inaccuracy behavior as *Lipschitz* embedding in **Euclidean space** for **tree-like** network topology. Also,

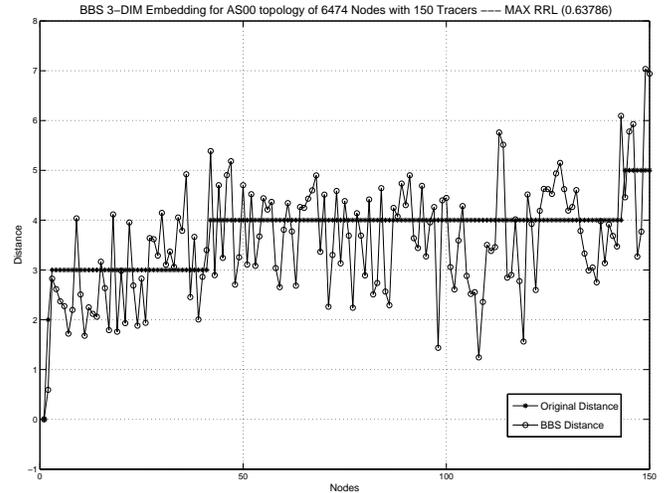
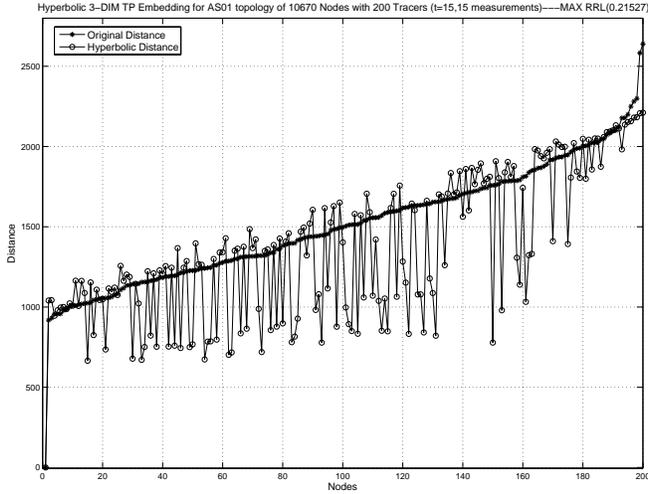


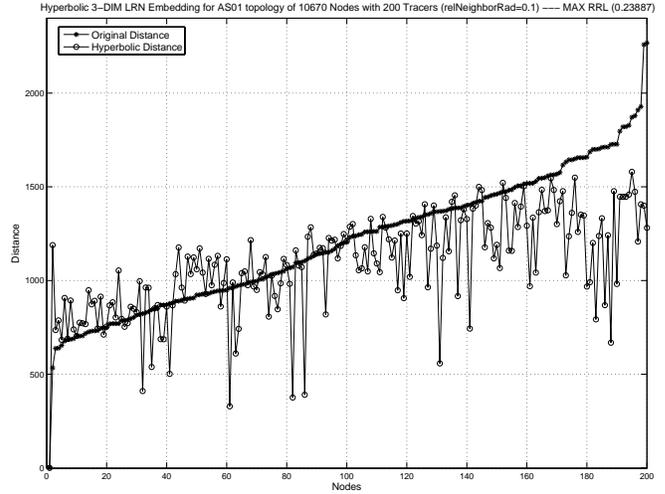
Figure 15: BBS (Euclidean) embedding using network topology of 150 nodes chosen from 6474 nodes in Jan 2000 AS Hierarchical Tree Network Topology — Node with Maximum rrl .

from the BBS (Hyperbolic) TP embedding experiment using BA Network Topology, it shows **similar** rrl inaccuracy behavior as the Lipschitz embedding using our PlanetLab site data. All these experiments show that the lists of close neighbors are being **pushed away** in the target Euclidean and Hyperbolic spaces.

We attempt to make a comparison study of BBS (Euclidean) and BBS (Hyperbolic) TP embeddings. The Jan 2000 AS Hierarchical Tree Network Topology (RouteViews) data set is used for this comparison. We examine



(a) Node with Maximum rrl for BBS (Hyperbolic) TP embedding.



(b) Node with Maximum rrl for BBS (Hyperbolic) LRN embedding.

Figure 16: rrl Comparison of BBS (Hyperbolic) TP embedding with 15 landmarks and 15 measurements and BBS (Hyperbolic) LRN embedding, using network topology of 200 nodes with lower degree chosen from 10670 nodes in Mar 2001 AS Hierarchical Tree Network Topology.

the signature plot of nodes with the maximum rrl for BBS (Hyperbolic) TP and BBS (Euclidean) embeddings, as illustrated in Figure 14(b) and Figure 15 respectively. For hierarchical tree-like network graphs, both embeddings in Euclidean and Hyperbolic spaces have **sharp bi-modal rrl** error behaviors. Similarly, in **Euclidean space**, the rrl measure for the BBS (Euclidean) embedding shows **similar rrl** inaccuracy behavior as the Lipschitz embedding from the perspective of the tree node in the synthetic binary trees, which we have illustrated in Section 3.5.

We continue to run rrl accuracy metric on BBS (Hyperbolic) LRN embedding using the March 2001 AS Hierarchical Tree Network Topology (RouteViews) data set. This is compared with BBS (Hyperbolic) TP embedding's parameters of $t = 15$ landmarks, and 15 node-to-landmark measurements for distortion minimization. Figure 16 shows the signature plots of the nodes with maximum rrl for BBS (Hyperbolic) TP and LRN embeddings. In LRN embedding, the list of close neighbors is being **pushed away very much further** and it has a **higher** maximum rrl compared to TP embedding. Both exhibit **similar bi-polar rrl** error behavior in Hyperbolic space.

7 Discussion

Our goal in this experimental work is to apply our new accuracy metrics to study the *accuracy* of estimated distances predicted by Internet coordinate systems and to what extent it varies among different pairs of nodes. The results of this initial attempt are not very encouraging. However, it remains that no single accuracy metric can capture the *qual-*

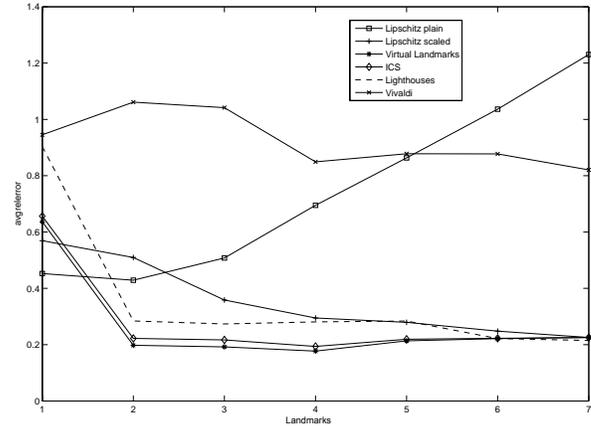


Figure 17: Average Absolute Relative Error Metric versus Number of Landmarks.

ity of the embedding techniques and it is worthwhile to develop a collection of accuracy metrics that are able to quantify user-oriented *quality*. Another interesting open problem is: Can we characterize the impact of network topologies that have good embeddings with respect to an accuracy metric? We envisage to answer this through our newfound future research on EONs and to discover its embeddability which aim to give good *quality* embeddings with respect to accuracy metrics. We also note that the number of landmarks has some impact on the accuracy of the embedding techniques: less information can give *better* accuracy results. Figures 17 and 18 show the accuracy metrics (average absolute relative error and rrl) versus number of land-

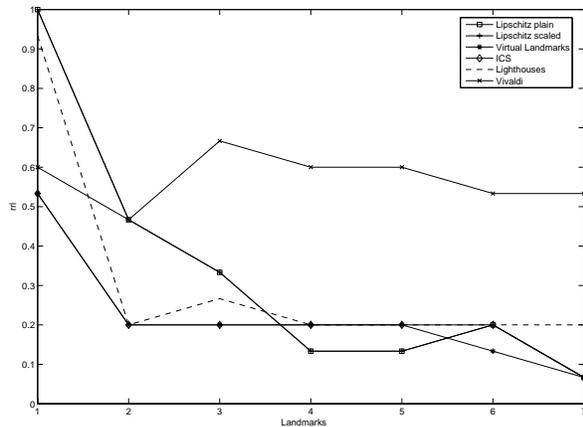


Figure 18: *rrl* Error Metric versus Number of Landmarks.

marks of the embedding techniques for the binary tree of depth 2 (Figure 1). The landmarks are selected sequentially and incrementally from the list of nodes in the binary tree. The results of this simple experiment indicate that Lipschitz plain embedding (without any scaling) gives *better* average relative absolute error and Vivaldi embedding gives *lower rrl* with small number of landmarks. Other embedding techniques have similar observation. This contradicts the intuition that having more landmarks (i.e. having more information) will give *better* accuracy results. A complete understanding will require the investigation of structural effects of the number of landmarks and landmarks' selection with respect to accuracy metrics.

8 Acknowledgments

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Notes

¹The all-pairs RTT measurement data was being continuously collected by Jeremy Stribling and it is available from http://www.pdos.lcs.mit.edu/~strib/pl_app/